Exercices en classe

frottement sec

forces généralisées

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PGI

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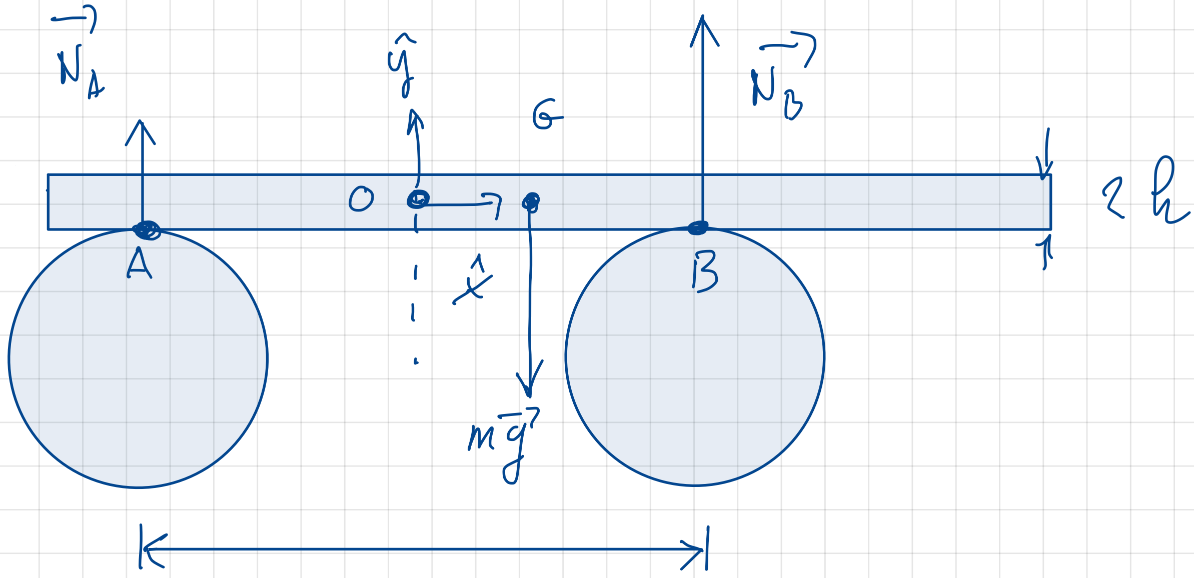


$$\|\vec{AB}\| \hat{=} d$$

$$\vec{OG} = x\,\hat{x}$$

$$\vec{OA} = -\frac{1}{2} d\,\hat{x} - h\,\hat{y}$$

$$\vec{OB} = +\frac{1}{2} d\,\hat{x} - h\,\hat{y}$$

$$\vec{M}_G = 0 = \vec{GA} \wedge \vec{N_A} + \vec{GB} \wedge \vec{N_B}$$

h est considéré comme petit et donc négligé.

$$\left[-\left(\frac{1}{2} d + x\right) N_A + \left(\frac{1}{2} d - x\right) N_B \right] \hat{z}$$

$$\frac{d+2x}{d-2x} = \frac{N_B}{N_A} \qquad (A)$$

Thm. centre de masse

$$m\,\ddot{x}\,\hat{x} = \vec{F}_A^{fr} + \vec{F}_B^{fr} = \mu_c\left(|N_A| - |N_B|\right)\hat{x}$$

$$m\,\ddot{x} = \mu_c\left(|N_A| - |N_B|\right) \qquad (*)$$

poutre en equilibre en y : $N_A\,\hat{y} + N_B\,\hat{y} - mg\,\hat{y} = 0 \qquad (B)$

(A) & (B)

$$|N_A| = \frac{mg}{2} - \frac{mg}{d} x$$

$$|N_B| = \frac{mg}{2} + \frac{mg}{d} x$$

(*) $\Longmapsto$

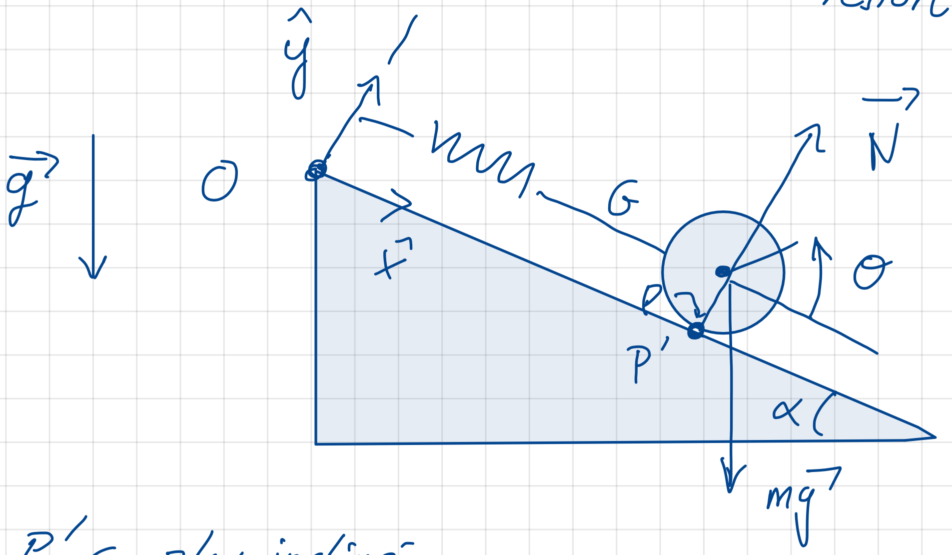
$$\boxed{\ddot{x} = -2 \frac{\mu_c g}{d} x}$$

application détermination de μ_c

$$x(t) = A \cos\left(\sqrt{\frac{2\mu_c g}{d}} \cdot t\right) + B \sin\left(\sqrt{\frac{2\mu_c g}{d}} \cdot t\right)$$

Forces généralisées

cylindre qui glisse sur plan incliné.
ressort + gravité



$P' \in$ plan incliné

$P \in$ cylindre $\qquad \vec{V}_{P'} = 0 \qquad \vec{V}_P \neq 0$

$$\vec{F_f} = -\mu_c \, ||\vec{N}|| \, \hat{V}_P$$

$$||\vec{N}|| = m g \cos\alpha$$

cinématique du solide

liaison

$$\vec{OG} = x\,\hat{x} + y\,\hat{y} \qquad y = +R$$

$$\vec{V}_P = \vec{V}_G + \vec{\omega} \wedge \vec{GP} \qquad \vec{\omega} = \dot{\theta}\,\hat{z}$$

$$\vec{GP} = -R\hat{y}$$

$$\vec{V}_G = \dot{\vec{OG}} = \dot{x}\,\hat{x}$$

$$\vec{V}_P = \dot{x}\,\hat{x} + \dot{\theta}\,\hat{z} \wedge (-R\hat{y})$$
$$= (\dot{x} + R\dot{\theta})\,\hat{x}$$

$$\vec{F_f} = -\mu_c \, \|\vec{N}\| \, \hat{v_p} \qquad \|\vec{N}\| = mg\cos\alpha$$

$$= -\mu_c \, mg\cos\alpha \; \text{sign}(\dot{x} + R\dot{\theta}) \, \hat{x}$$

$$\delta W = \vec{F_f} \cdot \delta\vec{OP}$$

$$\vec{OP} = \vec{OG} + \vec{GP}$$

$$\theta = \frac{3\pi}{2} \qquad P = P' \qquad \vec{GP} = R\cos\theta \, \hat{x} + R\sin\theta \, \hat{y}$$

$$\vec{OP} = x\,\hat{x} + R\cos\theta \, \hat{x} + R\sin\theta \, \hat{y}$$

$$\delta\vec{OP} = \delta x \, \hat{x} - R\sin\theta \, \delta\theta \, \hat{x} + R\cos\theta \, \delta\theta \, \hat{y}$$

$$P = P' \Rightarrow \theta = \frac{3\pi}{2} \qquad \sin\left(\frac{3\pi}{2}\right) = -1$$

$$\cos\left(\frac{3\pi}{2}\right) = 0$$

$$\delta\vec{OP} = \delta x \, \hat{x} + R \; \delta\theta \, \hat{x} + 0\hat{y}$$

produit scalaire

$$\delta W = \vec{F_f} \cdot \delta\vec{OP}$$

$$= \left(-\mu_c \, mg\cos\alpha \; \text{sign}(\dot{x} + R\dot{\theta})\right) \hat{x} \cdot \left(\delta x \, \hat{x} + R \, \delta\theta \, \hat{x}\right)$$

$$= \boxed{Q_x} \, \delta x + \boxed{Q_\theta} \, \delta\theta$$

$$\delta W = \left(-\mu_c \, mg \cos\alpha \;\; \mathrm{sign}(\dot{x} + R\dot{\theta})\right)\delta x$$
$$+ \left(-R \, \mu_c \, mg \cos\alpha \;\; \mathrm{sign}(\dot{x} + R\dot{\theta})\right)\delta\theta$$

$$\begin{cases} \dfrac{d}{dt}\left(\dfrac{\partial \mathcal{L}}{\partial \dot{x}}\right) - \dfrac{\partial \mathcal{L}}{\partial x} = Q_x \\[2ex] \dfrac{d}{dt}\left(\dfrac{\partial \mathcal{L}}{\partial \dot{\theta}}\right) - \dfrac{\partial \mathcal{L}}{\partial \theta} = Q_\theta \end{cases}$$